STEADY STATE FINITE ELEMENT ANALYSIS OF A SINGLE STACK COLD PLATE WITH HEAT LOSSES

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ABSTRACT

Steady state analysis of a single stack cold plate used for the cooling of electronic components is carried out using the finite element method. The present methodology takes into account the heat losses from the top and bottom surfaces of the stack. In addition dimensionless parameters are used in the analysis. The analysis is divided into two parts: a single unit cell analysis and the analysis of the assembly of several unit cells. The results from the present analysis of a single unit cell for single stack cold plate without heat losses compare well with those available in the literature. The analyses of the assembly of unit cells with heat losses from the top and bottom surface of the stack show that the single unit cell can be considered to be the representative of the stacks only when there are no heat losses.

1. INTRODUCTION

Cold plates are used to cool electronic components mounted on printed circuit board. A cold plate consists of an array of rectangular fins, attached between two exterior plates. Fluid passes through the spaces between the fins, to help increase the rate of heat transfer from the fin. A cold plate is called a single stack cold plate if there is no splitter plate between the two exterior plates. Steady state analysis of a cold plate was reported by Kraus¹ who provided an expression for its efficiency for the case of heat loading on one side. Kraus² then attempted to provide a weight optimisation of the cold plate with rectangular profile fins. Kern and Kraus³ analysed the single stack cold plate with heat input on one side, as well as both sides of the cold plate without heat losses from the top and bottom surfaces.

A stack is a combination of fins connected together. Thus, the whole stack is made of repeating arrays of fins. Kraus *et al.*⁴ showed that, for arrays of single fins, the conditions of heat flow and excess temperature at any point on a fin are influenced by similar conditions at the fin base. They abandoned the use of the conventional fin efficiency and proposed to express single fins and fin arrays by an important parameter called the thermal transmission ratio that was defined as the ratio of the heat entering the fin (or fin array) to the temperature excess at the base of the fin (or fin array). Later, this parameter was called the fin or array input admittance, defined at the fin

The variation of temperature along the fin is assumed to be linear as:

$$\theta = [N]\{\theta\} \quad (7)$$

where,

$$[N] = [1 - X \ X] \quad (8)$$

$$\{\theta\} = \begin{Bmatrix} \theta_i \\ \theta_j \end{Bmatrix} \quad (9)$$

θ_i, θ_j are dimensionless temperatures at nodes i and j respectively.

By using Galerkin's method, as explained in Segerlind⁷ and Lewis *et al.*⁸, the finite element formulation of equation (6) is obtained as

$$[K]\{\theta\} = \{0\} \quad (10)$$

where,

$$[K] = \begin{bmatrix} 1 + \frac{M}{3} & -1 + \frac{M}{6} \\ -1 + \frac{M}{6} & 1 + \frac{M}{3} \end{bmatrix} \quad (11)$$

The above theory is applied to the fin array and single stack cold plate. The details of the assembly of the element matrices for each case are given in the following section.

3. FIN ARRAY

In order to verify the present approach, we consider the case of a fin array for which solutions are available. Mikhailov and Ozisik⁹ modelled a fin array using a linear combination of two fundamental solutions to the governing differential equation for the one-dimensional steady state problem. However, in the present analysis, each fin can be considered to have more than one element. Thus the present finite element analysis is more general and can be used for longer fins as well.

Considering four elements of the fin array, as shown in figure 2(a), the element matrix for element 1 is written as:

$$\begin{bmatrix} 1 + \frac{M_1}{3} & -1 + \frac{M_1}{6} \\ -1 + \frac{M_1}{6} & 1 + \frac{M_1}{3} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (12)$$

where,

$$M_1 = \frac{hP_1B_1^2}{kA_1} \quad (5a)$$

P_1 = perimeter of fin 1, m

B_1 = width of fin 1, m

A_1 = cross-sectional area of fin 1, m².

Similarly, the element matrices for elements 2, 3 and 4 are as follows:

$$\begin{bmatrix} 1 + \frac{M_2}{3} & -1 + \frac{M_2}{6} \\ -1 + \frac{M_2}{6} & 1 + \frac{M_2}{3} \end{bmatrix} \begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (13)$$

$$\begin{bmatrix} 1 + \frac{M_3}{3} & -1 + \frac{M_3}{6} \\ -1 + \frac{M_3}{6} & 1 + \frac{M_3}{3} \end{bmatrix} \begin{Bmatrix} \theta_3 \\ \theta_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (14)$$

$$\begin{bmatrix} 1 + \frac{M_4}{3} & -1 + \frac{M_4}{6} \\ -1 + \frac{M_4}{6} & 1 + \frac{M_4}{3} \end{bmatrix} \begin{Bmatrix} \theta_3 \\ \theta_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (15)$$

where, M_2 , M_3 and M_4 are defined as in equation 5(a) for fins 2, 3 and 4, respectively.

Figure 2: (a) Geometry and dimensions of a fin array (Mikhailov and Ozisik⁹), (b) Finite element representation of a fin array.

The global matrix is an assembly of the element matrices. After incorporating the heat loadings at nodes 1 and 5, equation (10) is written as follows:

$$\begin{bmatrix} \left(1 + \frac{M_1}{3}\right) & 0 & \left(-1 + \frac{M_1}{6}\right) & 0 & 0 \\ 0 & \left(1 + \frac{M_2}{3}\right) & \left(-1 + \frac{M_2}{6}\right) & 0 & 0 \\ \left(-1 + \frac{M_1}{6}\right) & \left(-1 + \frac{M_2}{6}\right) & \left(4 + \frac{M_1}{3} + \frac{M_2}{3} + \frac{M_3}{3} + \frac{M_4}{3}\right) & \left(-1 + \frac{M_3}{6}\right) & \left(-1 + \frac{M_4}{6}\right) \\ 0 & 0 & \left(-1 + \frac{M_3}{6}\right) & \left(1 + \frac{M_3}{3}\right) & 0 \\ 0 & 0 & \left(-1 + \frac{M_4}{6}\right) & 0 & \left(1 + \frac{M_4}{3}\right) \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{Bmatrix} = \begin{Bmatrix} q_1 \\ 0 \\ 0 \\ 0 \\ q_5 \end{Bmatrix} \quad (16)$$

Table 1 shows the results of the present analysis considering eight elements as well as those from Mikhailov and Ozisik⁹. Mikhailov and Ozisik⁹ used excess temperature, which is equal to the actual nodal temperature minus the ambient temperature. These excess temperatures are converted to the non-dimensional values and are shown in table 1 for comparison.

It is clear from table 1 that there is a close agreement between the results of Mikhailov and Ozisik⁹ and the present analysis. This confirms that the present approach is valid for the assembly of fins.

4. SINGLE STACK COLD PLATE

A single unit cell, being the repeating segments of the stack, is considered first. The analysis of this single unit cell is carried out under the same operating conditions as given by Pieper and Kraus¹⁰. Next, different numbers of unit cells are considered and analysed after having assembled them together vertically with and without heat losses from the top and the bottom of the assembled unit. Results from the present analysis are then compared to those of Pieper and Kraus¹⁰ and then conclusions are drawn.

Table 1: A comparison of steady state excess temperature.

Node	Excess temperature in °C (Mikhailov and Ozisik ⁹)	Dimensionless temperature, θ from present analysis	Excess temperature in °C from present analysis
1	11.20	2.36	11.16
2	9.77	2.06	9.74
3	9.78	2.06	9.76
4	9.77	2.06	9.74
5	10.76	2.27	10.72

4.1. Analysis of a single unit cell

Pieper and Kraus¹⁰ analysed the single stack cold plate under the steady state conditions without heat losses from the top and the bottom surface of the stack. The geometry of a single unit cell is given in figure 3(a). When the present methodology is applied to a single unit cell having discretised into six one-dimensional fin elements as shown in figure 3(b), the global matrix is given in equation (17).

$$\begin{bmatrix} \left(1 + \frac{M_1}{3}\right) & \left(-1 + \frac{M_1}{6}\right) & 0 & 0 & 0 & 0 \\ \left(-1 + \frac{M_1}{6}\right) \left(3 + \frac{M_1}{3} + \frac{M_2}{3} + \frac{M_3}{3}\right) \left(-1 + \frac{M_2}{6}\right) & \left(-1 + \frac{M_1}{6}\right) & 0 & 0 & 0 & 0 \\ 0 & \left(-1 + \frac{M_2}{6}\right) \left(1 + \frac{M_2}{3}\right) & 0 & 0 & 0 & 0 \\ 0 & \left(-1 + \frac{M_2}{6}\right) & 0 \left(2 + \frac{M_3}{3} + \frac{M_4}{3}\right) & 0 & \left(-1 + \frac{M_4}{6}\right) & 0 \\ 0 & 0 & 0 & 0 \left(1 + \frac{M_5}{3}\right) & \left(-1 + \frac{M_5}{6}\right) & 0 \\ 0 & 0 & 0 & \left(-1 + \frac{M_4}{6}\right) \left(-1 + \frac{M_5}{6}\right) \left(3 + \frac{M_4}{3} + \frac{M_5}{3} + \frac{M_6}{3}\right) \left(-1 + \frac{M_6}{6}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \left(-1 + \frac{M_6}{6}\right) \left(1 + \frac{M_6}{3}\right) \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \end{Bmatrix} = \begin{Bmatrix} 1 + (\text{NUC} \times 4) \\ 1 + (\text{NUC} \times 2) \\ 1 + (\text{NUC} \times 4) \\ 0 \\ \bar{Q} + (\text{NUC} \times 4) \\ \bar{Q} + (\text{NUC} \times 2) \\ \bar{Q} + (\text{NUC} \times 4) \end{Bmatrix} \quad (17)$$

Figure 3: (a) Geometry of a unit cell of a single stack cold plate, (b) Finite element representation of a unit cell of a single stack cold plate, (c) Geometry of the assembly of 5 unit cells of a single stack cold plate, (d) Finite element representation of the assembly of 5 unit cells of a single stack cold plate.

Figure 4: Single stack cold plate: dimensionless temperature versus heat loading for a single cell analysis.

Figure 5: Single stack cold plate: temperature distribution at the left exterior plate for various NUC.

It may be mentioned here that the heat loading per unit cell on either side of the exterior plates are divided equally at the nodes of the elements considered. The heat loading for the whole stack at the left exterior plate is Q_{TL} , while the same is represented by Q_{TR} for the right exterior plate. The heat loadings are non-dimensionalised as given in equation (2). The reference dimension in equation (3) is the total width of the stack. NUC is used as the abbreviation for the number of unit cells. $NUC = 44.4$ and $M = 0.55$ for the case considered in Pieper and Kraus¹⁰. The results obtained can then be compared to those of Pieper and Kraus¹⁰, who presented the results of excess temperatures for a single stack cold plate for different heat loadings on the right exterior plate and a constant heat loading on the left exterior plate.

Figure 4 shows the maximum dimensionless temperatures on the left (θ_l) and right (θ_r) exterior plates. The highest value among the three nodes on each exterior plate is taken as the representative value. The results of Pieper and Kraus¹⁰ are also shown in figure 4. There is a good agreement of results between the present analysis and the results reported by Pieper and Kraus¹⁰. It shows that the present analysis applied to the single stack cold plate is correct.

4.2. Analysis of stack with varying NUC

Pieper and Kraus¹⁰ assumed that if the cold plate is made up of more than 20 repeating segments, a single segment is adequate for the purpose of analysis. Moreover, they neglected the edge effect of the stack. In the present analysis, several unit cells of a single stack cold plate are assembled in the vertical direction as in figure 3(c) and analysed for the same operating conditions. The purpose of analysing the assembled unit cells is to determine whether the assumption made by Pieper and Kraus¹⁰ regarding the analysis based on a single cell is correct. Furthermore, the edge effect is brought into account by allowing heat losses from the top and bottom surface of the stack.

Analyses of the assembled unit cells of varying number with and without heat losses are carried out. The heat loss is defined as a percentage of the total heat loading at the left exterior plate Q_{TL} . Since Q_{TL} increases proportionally with NUC, the heat loss also increases accordingly. This also implies that a heat loss of 10% for NUC = 20 is twenty times larger than a heat loss of 10% for NUC = 1. This definition of heat loss also implies that for a fixed NUC, a heat loss of 10% is the same irrespective of $\tilde{Q}$.

Figure 5 shows different curves of the temperature distribution at the left exterior plate for NUC = 1, 5, 20, 50 and 100 with heat loss (HL) from both ends = 0, 0.1 and 0.2 and $\tilde{Q} = 1$. In figure 5, notations like (HL0)₁, (HL1)₅, ... (HL2)₁, (HL2)₅ etc are used where HL stands for the heat loss, the number after HL, (0, 1, 2) represents zero loss, 0.1 (10%) loss and 0.2 (20%) loss respectively and the subscripts refer to the number of unit cells (NUC) considered for the analysis. It is observed from the figure that when there is no heat loss from both the ends, the temperature distribution is uniform throughout, irrespective of NUC, as shown by the curves (HL0)₁, (HL0)₅, (HL0)₂₀, (HL0)₅₀ and (HL0)₁₀₀. This justifies the assumption made by Pieper and Kraus¹⁰ that a single unit cell is a representative of the stack and is adequate for the purpose of analysis if there is no heat loss. When the heat loss equals 0.1, the maximum temperature θ_i obtained for NUC = 1 drops as compared to the earlier case (zero heat loss) as can be observed from the curve (HL1)₁. Results obtained for NUC = 5 and HL = 0.1 show that there is not much difference in θ_i as compared to that obtained for NUC = 1 and HL = 0.1 (curves (HL1)₁ and (HL1)₅). The same trend is observed when calculations are carried out for HL = 0.2. When NUC = 20, the value of θ_i is higher than that obtained for NUC = 1 or 5 when both heat losses are considered. Similar behaviour is observed when NUC is increased to 50. The lower temperatures at the ends and the symmetrical temperature distributions when the heat loss is considered are clearly shown by the different curves in figure 5. However, analyses with NUC = 100 and HL = 0.1 and 0.2 reveal that the temperatures of the near middle cells are closer to those obtained for the case with no heat loss as can be seen from the curves (HL1)₁₀₀, (HL2)₁₀₀, (HL0)₁, (HL0)₅, ... (HL0)₁₀₀. Furthermore, the curves (HL1)₁₀₀ and (HL2)₁₀₀ representing the temperature variation along the left exterior plate for different heat losses are very close to each other near the middle cells and differ at other locations. These results are not far from expectation. From the above analysis, it can be concluded that for large NUC (≥ 100) with heat

losses taking place, the analysis with a single unit cell at the middle without heat loss is adequate to get θ_p , which will be helpful to determine whether the maximum temperature limit θ_{max} has been achieved or not. The above analysis also shows that a single unit cell analysis with heat loss does not represent the conditions of the middle cell of a stack having any number of repeating segments of unit cell.

The temperature distributions along the right exterior plate for different values of NUC and different heat losses are the same as those on the left exterior plate when $\tilde{Q}=1$. When $\tilde{Q}$ is less than 1, the left exterior plate will have temperature levels higher than those of right exterior plate. This behaviour is shown in figure 6, where results are plotted for $M = 0.55$, $NUC = 20$ and heat loss = 0.2. Similarly, when $\tilde{Q}$ is greater than 1, the opposite behaviour is expected and shown in the figure for $\tilde{Q} = 2$.

Temperature distributions along the horizontal fin of the middle unit cell are shown in figure 7 where heat loss equals to 0.2 with different NUC and different $\tilde{Q}$. Results with no heat loss for all heat loadings considered are also shown in figure 7. The symmetric temperature distribution for $\tilde{Q}=1$ is clearly observed in the figure. When $\tilde{Q}=0.33$, the temperature at the right nodes is lower than that at the left node, which is expected, and the trend is reversed for $\tilde{Q}=2$. It is seen that for any $\tilde{Q}$, as NUC increases, the temperature

Figure 6: Single stack cold plate: temperature distribution at the left and right exterior plates for different $\tilde{Q}$.

Figure 7: Single stack cold plate: temperature distribution along the horizontal fin at the middle of the assembled stack.

distribution approaches the case where there is no heat loss. The temperature distribution with $NUC = 100$ for all $\tilde{Q}$ coincides with that when there is no heat loss from both the ends of the assembled unit cells. This shows that for $NUC \geq 100$, the effect of heat loss does not affect the cells at or near the middle unit cell. The above results further strengthen the point that a single unit cell analysis of a cold plate does not reflect the true conditions of the cold plate if losses are taken into account, which happens in the actual situation.

All the above results and discussions were limited to a particular value of the dimensionless parameter $M = 0.55$ for a single stack cold plate, which corresponds to the case of Pieper and Kraus¹⁰. In order to present a generalized behaviour of the cold plate, analyses for different values of M are carried out with and without heat losses from the top and bottom surfaces of the stack. These analyses have generated new and additional data, which will be helpful in the design of the cold plate used for the cooling of the electronic systems. The results are plotted in terms of the maximum dimensionless temperature on the left exterior plate θ_l for different values of M and for two different values of heat loading $\tilde{Q}$ when the heat loss = 0.2 as shown in figure 8. The effect of the variation of NUC is also shown in the figure. Calculations are also carried out for no heat loss and the results are plotted in the same figure. Thus, figure 8 represents a generalized curve applicable for any single stack cold plate for $\tilde{Q} = 1$ & 2 with a heat loss = 0.2. In general, θ_l for $\tilde{Q} = 2$ is higher than that for $\tilde{Q} = 1$ for all the values of $0.25 < M < 2$. For lower values of M , the difference between the values of θ_l for the two values of $\tilde{Q}$ is large but this difference reduces for $M > 0.5$. A single point on each curve, corresponding to zero heat loss for the two heat loadings, refers to the result presented by Pieper and Kraus¹⁰.

Figure 8: Single stack cold plate: θ_l versus M .

Figure 9: Single stack cold plate: θ_l and θ_r versus M .

Figure 10: Single stack cold plate: temperature distribution at the left and right exterior plate.

It is also noticed from figure 8 that for $\tilde{Q} = 2$ and $NUC = 50$, θ_i values for $M > 0.75$ coincide with those calculated for no heat loss. Similar observation is noticed from the lower curves for $\tilde{Q} = 1$ but beyond a slightly higher value of M as compared to that established for $\tilde{Q} = 2$. This suggests that for values of $M \geq 0.75$ and even with a heat loss of 0.2, the results can be obtained from the analysis of a single unit cell without heat loss when NUC exceeds 50.

Figure 9 shows the maximum temperature at the left and right exterior plate (θ_l and θ_r) plotted against M for different $\tilde{Q}$ at heat loss = 0.2 and $NUC = 100$. When $\tilde{Q} = 0.33$, temperature at the left exterior plate is generally higher than that at the right exterior plate for any values of M . When $\tilde{Q} = 1$, temperature is the same at both the plates, but when $\tilde{Q} = 2$, the right exterior plate shows higher temperature.

It is also possible to calculate the temperature distribution at the left and right exterior plates if the heat losses from the top and bottom of the assembled unit cells are different as happens in the actual situation. Figure 10 shows such results for the case where $NUC = 20$, $\tilde{Q} = 0.33$, heat loss from the top is 0.2, while heat loss from the bottom is varied from 0 to 0.2. When the heat loss at the top is larger than at the bottom, temperature drop at the top of the cold plate is more than the bottom. When the heat losses at both ends are the same, temperature distribution is symmetric about the middle of the stack.

5. CONCLUSIONS

The finite element method is used to analyse a single stack cold plate with heat losses under steady state conditions. A single one-dimensional fin theory is applied to the discretised elements in the above analysis. The results of the generalized analysis of single stack cold plates for different values of M , a dimensionless parameter involving

dimension, properties of the stack material and h are also presented, which will be helpful in the design of cold plates used for the cooling of electronic systems. The following conclusions are drawn:

- (i) The results obtained from the analysis of a single unit cell without heat losses from the top and bottom surfaces agree well with those available in the literature. Thus, the single unit cell can be taken as the representative of the single stack cold plate only when there are no heat losses from the top and bottom surfaces of the stack.
- (ii) When heat losses from the top and bottom surfaces of the single stack cold plate consisting of less than 100 NUC are to be considered, the analysis of the whole stack should be carried out to determine the maximum temperature of the right or left exterior plate for any heat loading. Analysis with a single unit cell being representative of the stack under this condition will give much lower value of the maximum temperature. However, the analysis of a single unit cell without heat loss will give the same result in terms of maximum temperature as for cold plates with $NUC > 100$ and heat loss from the top and bottom surfaces taking place.
- (iii) New results in terms of the maximum temperature of the stack obtained under different heat loadings for various values of M are presented. Analysis of the single stack cold plate having $M > 0.75$ with $NUC = 50$ and heat loss being considered give the same result as that obtained from a single unit cell without heat loss for the same heat loading. Thus, for higher values of M , $NUC = 50$ is sufficient for the analysis to get the maximum temperature.
- (iv) New results in terms of the temperature distribution along the left and right exterior plates for heat losses from the top surface being different from the bottom surface for a particular heat loading and a particular value of M are also presented. When the heat loss at the top is larger than at the bottom, temperature drop at the top of the cold plate is more than that at the bottom. When the heat losses at both ends are the same, temperature distribution is symmetric about the middle of the stack.

6. REFERENCES

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