Evaluation of Mixture Viscosity Models in the Prediction of Two-phase Flow Pressure Drops

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Nine existing mixture viscosity models were tested for predicting a two-phase pressure drop for oil-water flow and refrigerant (R.134a) flow. The predicted data calculated by using these mixture viscosity models were compared with experimental data. Predicted data from using one group of mixture viscosity models had a good agreement with the experimental data for oil-water two-phase flow. Another group of viscosity models was preferable for gas-liquid flow, but these models gave underestimated values with an error of about 50%. A new and more reliable mixture viscosity model was proposed for use in the prediction of pressure drop in gas-liquid two-phase flow.

Key words: Mixture viscosity model; prediction; oil-water, gas-liquid two-phase flow; pressure drop; refrigerant R.134a

Over the years, studies on two-phase (gas-liquid, liquid-liquid) flows have accelerated because understanding the two-phase flow phenomenon becomes more and more important and necessary in scientific and industrial applications. In many works of research, the characteristics of two-phase flow have been investigated by using theoretical and experimental approaches. Numerous theoretical models and empirical correlations have also been developed for predicting two-phase pressure drop. However, no general model reliably predicts the two-phase flow pressure drop because of the complexity of phase interaction, motion and deformation of the interface between phases, as well as non-linearity characteristics. Therefore, by trial and error, many researchers are developing new models and testing these for predicting two-phase flow pressure drop in attempts to improve the accuracy of prediction.

The homogenous model is one of the oldest models and provides the simplest technique

for analyzing two-phase (or multiphase) flows. The homogeneous model considers two-phase flow as a single-phase flow having average fluid properties which depend upon mixture quality. Here, many averaging methods have been used to find the mixture properties (*especially mixture viscosity*) which strongly influence two-phase flow pressure drop. In a literature review, it was found that many of the mixture viscosity models were developed for use in a homogeneous two-phase flow model. Using different mixture viscosity models will result in different predictions. Although mixture viscosity models (or mixture viscosity correlations) have been extensively developed in the past, little attention has been paid to the comparison of these various models. Somchai Wongwises (2008) examined the applicability of several widely used viscosity models to pressure drop prediction in air-water flow through a 0.53 mm diameter micro channel. Awad (2008) examined the error of existing viscosity models in predicting pressure drop for various refrigerants (R12, R22, R740,

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R717, etc.) in micro and mini channels, and he proposed new definitions for two-phase mixture viscosity by using an analog between thermal conductivity. The prediction of pressure drop using the homogeneous model is reasonably accurate only for bubble and mist flows because the entrained phase travels at nearly the same velocity as the continuous phase. Besides, if the density ratio (ρ_L/ρ_G) approaches 1 (i.e. nearly equal densities), the homogeneous model is also applicable for predicting pressure drop.

Previous work has studied viscosity models only for air-water two-phase flow and refrigerant two-phase flow in micro and mini channels. In pipeline-conveying systems, one of the most common but least understood phenomenon is the mixing behaviour of two immiscible liquids, especially oil and water. Accurate prediction of the viscosity of an oil-water mixture is essentially needed for the calculation of frictional pressure drop for pipeline design purposes. In addition, the reliability level of some existing mixture viscosity models should be examined, and these should be categorized in their use for liquid-liquid flow and gas-liquid flow. Some attempts have been made to compare reliabilities of mixture viscosity models for some types of refrigerants but not for R134a.

Therefore, the objective of this work is to investigate the influence of mixture viscosity models in the prediction of pressure drop for both liquid-liquid two-phase flow (oil-water) and gas-liquid two-phase flow (refrigerant, R134a) in micro-, mini- and large-diameter circular pipes.

HOMOGENEOUS TWO-PHASE FLOW MODEL

In the homogeneous model, the fluids are characterized by a fluid that effectively has suitably averaged properties of the two phases in the pipe. This is also known as the *No-Slip Model*. It is assumed that there is no velocity difference between the phases and so the two fluids flow at the same velocity.

By assuming no heat and mass transfer for the homogeneous model, the pressure gradient can be written as the sum of three pressure gradient components, namely, due to friction, gravity and acceleration:

$$-\left(\frac{dp}{dz}\right)_{\text{total}} = 2f_h \rho_h \frac{U_m^2}{2} + \rho_h g \sin(\theta) + \frac{d}{dx} \left[\rho_G \frac{U_{SG}^2}{\alpha_G} + \rho_L \frac{U_{SL}^2}{\alpha_L} \right] \quad (1)$$

Normally, the contribution of momentum pressure drop is very small. To simplify the theory in analytical predictions, momentum pressure drop in *Equation 1* will be ignored, and, for horizontal flow, pressure drop by elevation difference is zero. The void fraction based on the homogeneous model (α_h) can be expressed as follows:

$$\alpha_h = \beta_G = \frac{Q_G}{Q_G + Q_L} = \frac{U_{SG}}{U_m} \quad (2)$$

The homogeneous void fraction can also be expressed in terms of mass quality (x):

$$\alpha_h = \frac{1}{1 + \left(\frac{1-x}{x}\right) \left(\frac{\rho_G}{\rho_L}\right)} \quad (3)$$

For the homogeneous model, the homogeneous density of two-phase flow (ρ_h) can be expressed as:

$$\rho_h = \alpha_G \rho_G + \alpha_L \rho_L \quad (4)$$

or it can be expressed in terms of mass quality (x):

$$\rho_h = \left(\frac{x}{\rho_G} + \frac{1-x}{\rho_L} \right) \quad (5)$$

The mixture viscosity is defined as:

$$U_m = U_{SG} + U_{SL} \quad (6)$$

In the above equations, the required basic parameters can be calculated as follows:

$$x = \frac{G_G}{G_{tot}} \quad (7)$$

$$U_{SG} = \frac{G_G}{\rho_G} \quad (8)$$

$$U_{SL} = \frac{G_{tot} - G_G}{\rho_L} \quad (9)$$

The Reynolds number based on the homogeneous model can be expressed as:

$$Re_h = \frac{D \rho_h U_m}{\mu_h} = \frac{G_{tot} D}{\mu_h} \quad (10)$$

The required Fanning friction factor can then be calculated from Blasius correlation:

$$\left. \begin{aligned} f &= \frac{16}{Re} & Re \leq 2300 \\ f &= \frac{0.079}{Re} & Re > 2300 \end{aligned} \right\} \quad (11)$$

where, μ_h in Equation 10 is two-phase mixture viscosity. Several two-phase viscosity models proposed by different researchers are given in Equations 12–20, shown below:

$$\text{Owen's model (1961)} \quad \mu_h = \mu_L \quad (12)$$

$$\text{McAdams } et al.'s \text{ model (1942)} \quad \frac{1}{\mu_h} = \frac{x}{\mu_G} + \frac{(1-x)}{\mu_L} \quad (13)$$

$$\text{Lin } et al.'s \text{ model (1991)} \quad \mu_h = \frac{\mu_L \mu_G}{\mu_G + x^{1.4} (\mu_L - \mu_G)} \quad (14)$$

$$\text{Cicchitti } et al.'s \text{ model (1960)} \quad \mu_h = x\mu_G + (1-x)\mu_L \quad (15)$$

$$\mu_h = \beta_G \mu_G + (1 - \beta_G) \mu_L \quad (16)$$

$$\mu_h = \beta_G \mu_G + (1 - \beta_G)(1 + 2.5\beta_G) \mu_L \quad (17)$$

$$\mu_h = \rho_h \left(\sqrt{x v_G} + \sqrt{(1-x) v_L} \right) \quad (18)$$

$$\mu_h = \mu_L \left[1 - x \left(\frac{\rho_L}{\rho_G} - 1 \right) \right] \quad (19)$$

$$\mu_h = \mu_L \left(\frac{\rho_h}{\rho_L} \right) \quad (20)$$

INVESTIGATION PROCEDURE

Predicted two-phase pressure drops were calculated by using the homogeneous two-phase flow model with variation of the mixture viscosity models from *Equations 13 to 20*. For oil-water two-phase flow, the experimental data were obtained from Angeli (2006). For refrigerant (R134a) two-phase flow, data on experimental pressure drop were obtained from Kattan (1996) and Thome (2006). Details on the experimental setup and strategies of measurements are given in (Angeli 2006), (Kattan 1996) and (Thome 2006). The required fluid properties were taken from the experiments and shown in *Tables 1, 2 and 3*.

To determine the effect of the mixture viscosity models, only frictional pressure drops from the experimental data were used to compare with the predicted data. Good agreement between the experimental data and predicted data when using the well-known viscosity models was defined by having a low root mean square (rms) error.

The error (ε) from applying a model to each available data point is defined as:

$$\varepsilon (\%) = \left| \frac{\text{Predicted} - \text{Experimental}}{\text{Experimental}} \right| \times 100 \quad (21)$$

For groups of data, the root mean square error, ε_{rms} , is defined as:

$$\varepsilon_{\text{rms}} = \left[\frac{1}{N} \sum_{k=1}^N \varepsilon_k^2 \right]^{1/2} \quad (22)$$

Table 1. Properties of oil and water at $T = 25^\circ\text{C}$.

Liquid	Oil Exxsol D140 (Exxon Chemicals)	Water
Density (kg/m^3)	828	998
Viscosity (N.s/m^2)	0.0055	0.000993

Table 2. Properties of refrigerant R134a at $T_{\text{sat}} = 4^\circ\text{C}$

Fluid	Refrigerant, R134a (Liquid)	Refrigerant, R134a (Vapour)
Density (kg/m^3)	1282	16.7
Viscosity (N.s/m^2)	0.000254	0.000011

Table 3. Properties of refrigerant R134a at $T_{\text{sat}} = 30^\circ\text{C}$

Fluid	Refrigerant, R134a (Liquid)	Refrigerant, R134a (Vapour)
Density (kg/m^3)	1188	37.5
Viscosity (N.s/m^2)	0.000181	0.0000119

RESULTS AND DISCUSSION

Oil-water Two-phase Flow

Comparisons of the predicted frictional pressure gradients and published experimental data are shown in *Figure 1* and *Figure 2*. The experimental data taken from Angeli (2006) frictional pressure gradients for a mixture velocity of 1.5 m/s and volumetric oil concentration between 10% and 90%. According to his definition, the mixture viscosity models can be classified into two groups. The first group [Group I] satisfies the following conditions:

$$\left. \begin{array}{ll} \text{if } x = 0 & \mu_h = \mu_L \\ \text{if } x = 0 & \mu_h = \mu_G \end{array} \right\}$$

Group I includes *Equations 13–18*.

The second group [Group II] includes *Equations 12, 19 and 20* which do not satisfy the above conditions.

The results of this work show that Group II mixture viscosity models gave fair predictions which had the same trends as the experimental data. In Group II, García *et al.*'s model (2003) had a good agreement with the experimental data. Group I mixture viscosity models has

different trends compared with the experimental data, i.e. Group I's predictions gave increasing values while the experimental data showed decreasing values with an increase in oil concentration.

Predictions by Group I models were theoretically correct. When the volumetric oil concentration becomes higher, the mixture viscosity will increase and that will raise the pressure gradient. However, the experimental data on pressure gradient decreased with an increase in oil concentration. Angeli (2006) said that the reason of the decreasing pressure gradient is because of high *in situ* water fraction at an upward inclination. Therefore the mixture viscosity is still close to water viscosity even when oil concentration becomes high. This is noteworthy for oil-water two-phase flows.

For liquid-liquid two-phase flow, García *et al.*'s model (2003) was the most appropriate one (*especially for upward inclined flow*). Unfortunately comparison of the predicted data and experimental friction pressure drops in horizontal oil-water flow could not be reported in this work. However, it is reasonable to say that the frictional drops in inclined flow and horizontal flow are not so different.

Figure 1. Predicted and experimental frictional pressure gradients for oil-water two-phase flow at $U_m = 1.5$ m/s, $+10^\circ$ inclination (Angeli 2006).

Figure 2. Predicted and experimental frictional pressure gradients for oil-water two-phase flow at $U_m = 1.5$ m/s, $+10^\circ$ inclination (Angeli 2006).

The ϵ_{rms} values of each model are shown in Table 4 in order of increasing error. It can be seen that for liquid-liquid flows with nearly the same densities (i.e. $\rho_L/\rho_G \approx 1$), Group II mixture viscosity models made good predictions. From Group I, Lin *et al.*'s model gave the smallest value of ϵ_{rms} .

From this study, it is shown that for liquid-liquid two-phase flows, making the assumption that the viscosity of the continuous phase is the same as the mixture viscosity can be considered reasonable until the medium range of dispersed phases' concentration.

Refrigerant (R134a) Flow

For refrigerant (R134a) two-phase flow, the experimental data from Kattan's work (1996) were obtained by performing experiments in 10.92 mm and 12 mm diameter circular horizontal pipes. In this study, comparisons were made with the experimental data for $D = 10.92$ mm and $G_{tot} = 300$ kg.s⁻¹.m⁻² with variations in vapour quality. The comparisons are shown in *Figure 3* and *Figure 4*. For gas-liquid flow, most of the mixture viscosity models gave underestimated values. All the mixture viscosity models in Group I had nearly the same predictions and ϵ_{rms} around 50%. For gas-liquid flow, Cicchitti *et al.*'s model (1960) and Fourar and Bories model (1995) gave

relatively better predictions although they had very high errors in their predictions for oil-water flow.

It can be observed that Davidson *et al.*'s model (1943) was not very suitable for predicting gas-liquid two-phase flow because it had a very high error term. From this definition, it can be seen that Davidson *et al.*'s model (1943) was suitable for two-fluid flow where $\rho_L/\rho_G \approx 1$. García *et al.*'s model (2003) gave very low predictions with an error of 66.64%. Therefore, Group II viscosity models (except Owen's model (1975)) are less preferable for pressure drop prediction in gas-liquid two-phase flows.

From this study, it is shown that it is convenient to use Owen's viscosity model (1961) in both liquid-liquid flow and gas-liquid flow because of it gives fair predictions for both types of flow. The ϵ_{rms} values from each viscosity model in predicting gas-liquid two-phase flow are shown in *Table 5*.

DEFINING A NEW MIXTURE VISCOSITY MODEL FOR GAS-LIQUID FLOW

In previous mixture viscosity models, the homogeneous viscosity decreases with increasing gas concentration. This gives a high error in estimation. Actually, at low gas concentrations, the viscosity of the gas has no

Table 4. ϵ_{rms} from each mixture viscosity model for oil-water flow.

No.	Mixture viscosity model	ϵ_{rms} (%)
1	García <i>et al.</i> 's model	20.3271
2	Owen's model	22.8621
3	Davidson <i>et al.</i> 's model	25.6263
4	Lin <i>et al.</i> 's model	37.8751
5	McAdams <i>et al.</i> 's model	42.7214
6	Cicchitti <i>et al.</i> 's model	63.0710
7	Dukler <i>et al.</i> 's model	64.9552
8	Beattie and Whalley's model	70.1044
9	Fourar and Bories' model	83.8308

Figure 3. Predicted and experimental frictional pressure gradients for R134a two-phase flow $G_{tot} = 300 \text{ kg.s}^{-1}.\text{m}^{-2}$, $D = 10.92 \text{ mm}$ (Kattan 1996).

Figure 4. Predicted and experimental frictional pressure gradients for R134a two-phase flow $G_{tot} = 300 \text{ kg.s}^{-1}.\text{m}^{-2}$, $D = 10.92 \text{ mm}$ (Kattan 1996).

significant contribution and the viscosity of the liquid is still dominant in the mixture. Thus, Owen's viscosity model ($\mu_h = \mu_L$) is an acceptable model for gas-liquid flow.

In fact, the homogenous flow model can reasonably be used for only bubbly flow and mist flow. Therefore, in bubbly flow, the effect of the turbulent characteristics induced by the bubbles should not be neglected. Kashinsky (1992) proved that laminar bubbly flow has turbulent characteristics and the wall shear stress becomes (1.5~3 times) higher than in single phase flow, depending on bubble size. Therefore, it is reasonable to define the new mixture viscosity model by including turbulent viscosity even if the flow is laminar.

$$\mu_h = \mu_L + \mu_T \quad (23)$$

Sato *et al.* (1975) introduced bubble-induced turbulent viscosity as follows:

$$\mu_T = C_{\mu} \rho_L \frac{k^2}{e} + C_{\mu b} \rho_L \alpha_G d_B \left| U_G - U_L \right| \quad (24)$$

By assuming homogeneous ($U_G = U_L$) flow, the second term of *Equation 24* is zero. Therefore,

$$\mu_h = \mu_L + C_{\mu} \rho_L \frac{k^2}{e} \quad (25)$$

According to the experimental data of Kashinsky (1992), $\tau_{W \text{ (two phase)}} = (1.5 \sim 3) \times \tau_{W \text{ (single phase)}}$ depending on bubble size; therefore, it is reasonable to make definitions that

$$\mu_h = (1.1 \sim 2) \mu_L \quad (26)$$

which is higher than Owen's viscosity model by including turbulent viscosity from 10% to 100% . The proposed mixture viscosity model was tested to predict the data of Kattan (1996). A comparison of the data is shown in *Figure 5*, while ϵ_{rms} values are given in *Table 6*.

Table 6. ϵ_{rms} from each mixture viscosity model for refrigerant R.134a flow.

No.	Mixture viscosity model	ϵ_{rms} (%)
1	Owen's model	28.294
2	Definition 1 ($\mu_h = 1.1 \mu_L$)	27.577
3	Definition 2 ($\mu_h = 1.3 \mu_L$)	26.572
4	Definition 3 ($\mu_h = 1.5 \mu_L$)	26.096
5	Definition 4 ($\mu_h = 1.7 \mu_L$)	26.024
6	Definition 5 ($\mu_h = 2 \mu_L$)	26.472

It can be seen that definition 4 from *Equation 26* gave the lowest ϵ_{rms} . However, *Equation 26* should be used in a limited range of gas concentrations because when the gas concentration becomes high, the viscosity of gas will be significantly involved in the mixture viscosity. Therefore, by choosing definition 4, it is desirable to define the new mixture viscosity model as follows:

$$\left. \begin{aligned} \mu_h &= 1.7\mu_L & (0 < x < 0.6) \\ \mu_h &= 1.7 [x\mu_G + (1 - x) \mu_L] & (x > 0.6) \end{aligned} \right\} \quad (27)$$

Figure 5. Predicted and experimental frictional pressure gradients for R134a two-phase flow $G_{tot} = 300 \text{ kg.s}^{-1}.\text{m}^{-2}$, $D = 10.92 \text{ mm}$ (Kattan 1996).

Figure 6 to Figure 8 show the comparison of the experimental data and predicted data using Owen's mixture viscosity model and the present viscosity model (Equation 27). The experimental data were taken from Kattan (1996) and Thome (2006). The data of Thome (2006) were obtained from conducting experiments for the refrigerant R134a at $T_{\text{sat}} = 30^\circ\text{C}$ in a micro channel with a diameter of 0.509 mm. It can be seen that the present mixture viscosity model gave the best predictions with the lowest ϵ_{rms} . This proves that defining the new viscosity model by taking into account turbulent viscosity is very reasonable and appropriate for gas-liquid flow. A comparison of the mixture viscosity results from some of the models are shown in Figure 6 to Figure 8. Figure 9 shows the trend of the ratio of mixture viscosity to liquid viscosity for each model according to mass quality. It can be seen that Owen's model behaved like a datum line, showing values above the datum line to be higher than liquid viscosity and those below the datum line lower than liquid viscosity.

CONCLUSION

A reliable and accurate prediction of mixture viscosity is crucial in the evaluation of pressure drop for piping system design. An underestimation can lead to an insufficiently powered design, while an overestimation can result in a higher cost for an over-powered design. Further impacts include breakdowns in operation and effects on operating costs. Thus, this study investigated the effectiveness of existing mixture viscosity models in two-phase flow pressure drop prediction. Frictional pressure drops of oil-water flow and refrigerant (R134a) two-phase flow in micro, mini and large diameter pipes were predicted by using the homogeneous model with variation of the mixture viscosity model. From the results of this study, the following conclusions were made:

- Of the two groups of mixture viscosity models, Group II is suitable for use in pressure drop predictions for liquid-liquid flow (i.e. $\rho_L/\rho_G \approx 1$). In this group, García

Figure 6. Predicted and experimental frictional pressure gradients for R134a two-phase flow $G_{tot} = 300 \text{ kg.s}^{-1}.\text{m}^{-2}$, $D = 10.92 \text{ mm}$ (Kattan 1996).

Figure 7. Predicted and experimental frictional pressure gradients for R134a two-phase flow $G_{tot} = 700 \text{ kg.s}^{-1}.\text{m}^{-2}$, $D = 0.509 \text{ mm}$ (Thome 2006).

Figure 8. Predicted and experimental frictional pressure gradients for R134a two-phase flow $G_{tot} = 1000 \text{ kg.s}^{-1}.\text{m}^{-2}$, $D = 0.509 \text{ mm}$ (Thome 2006).

Figure 9. Comparison of mixture viscosity results from various viscosity models.

et al.'s model gives the best predictions in oil-water two-phase flow, but it has a very high error term in gas-liquid two-phase flow.

- Most of mixture viscosity models give underestimated values in predicting gas-liquid two-phase pressure drop. Except for Owen's model, Group II mixture viscosity models are not appropriate for pressure drop prediction in gas-liquid two-phase flow.
- Owen's viscosity model gives fair predictions in both liquid-liquid flow and gas-liquid flow.
- Most of existing mixture viscosity models assume that the mixture viscosity will decrease with increasing gas concentration. That is why these viscosity models give underestimated predictions. At low gas concentrations, the viscosity of gas will not contribute significantly to the mixture viscosity. Moreover, gas bubbles will create turbulent viscosity that will increase the mixture viscosity.
- The present mixture viscosity model is defined by including turbulent viscosity. It shows the best predictions in gas-liquid two-phase flow.
- The present mixture viscosity model can be strongly recommended for giving the best predictions for gas-liquid (which have nearly the same physical properties as R134a) two-phase flow at low gas concentrations.

ACKNOWLEDGEMENT

The author would like to express special thanks to his supervisor and advisor, Prof Ir. Triyogi Yuwono, DEA, for his valuable advice, patience, understanding and intellectual challenges throughout the duration of this research, and to all the members of the Fluid Mechanic Laboratory, Mesin, ITS for their support in providing facilities and great help.

Date of submission: October 2012

Date of acceptance: February 2013

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NOMENCLATURE

Notations

C_μ	Constant in turbulent model
D	Pipe diameter (m)
d	Bubble diameter (m)
e	Turbulent dissipation rate (m^3/s)
f	Fanning friction factor
G	Mass flux ($\text{kg}/\text{m}^2\text{s}$)
k	Turbulent kinetic energy (m^2/s^2)
P	Pressure (N/m^2)
Q	Flow rate (m^3/s)
Re	Reynolds number
U	Velocity (m/s)
x	mass quality
Z	Pipe length along horizontal axis (m)

Greek Letters

α	Phase fraction
μ	Absolute viscosity (Ns/m^2)
ν	Dynamic viscosity (m^2/s)

β	Phase concentration
ϵ	Relative error
ρ	Density (kg/m^3)
θ	Inclination angle (degree)
τ	Shear stress (N/m^2)

Subscripts

B	bubble
f	friction
G	gas
h	homogeneous
L	liquid
m	mixture
SG	superficial gas
SL	superficial liquid
tot	total
T	turbulent
W	wall